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Local Lagrangian quantum field theory of electric and magnetic charges of spin zero mesons

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We present a local Lagrangian field theory of spin zero mesons carying both electric and magnetic charges. The quantization of $(e_m g_n - e_n g_m)$ as integral multiples of 4π is obtained as a condition of integrability of infinitesimal Poincaré-Lie algebra to finite Poincaré group. Canonical commutation relations, energy momentum tensor and Feynman rules are obtained by usual techniques of Lagrangian field theory.

1. Introduction

There is an inherent asymmetry in Maxwell-Lorentz electrodynamics in the sense that this theory incorporates free electric charges but not free magnetic charges. This is in agreement with experimental observation that there does not exist free magnetic charges in Nature. But actually nothing in Classical Physics forbids the existence of free magnetic charges To remedy this lack of summetry Dirac (1931, 1948) reformulated Maxwell-Lorentz theory so as to include magnetic charge and showed that quantisation is possible provided only the charges and pole strengths have to be quantized according to $eg/hc = n/2$ Schwinger (1966) proposed that charge quantization is given by $eg/hc = n$ which makes smallest non-zero charge-pole product unity instead of 1/2 as was shown by Diracs's quantization condition This integral quantization is due to the use of an infinite discontinuity line in accordance with space reflection consideration rather than semi-infinite line employed by Dirac. Both Dirac and Schwinger formulated the theories of magnetic charges carried by spin-1/2 fields. Schwinger's field theory of magnetic charge was extended to the case of charged fields with zero and one spin by Yan (1966). Zwanziger (1968) has developed the field theory of magnetic charges of spin-1/2 fields, the particles carrying both electric and magnetic charges. He showed the unitary equivalence of Hamiltonians describing the system of particles with electric and magnetic charges e_n, g_n and the system with charges $e_n = e_n \cos\theta + g_n \sin\theta$, $g_n = -e_n \sin\theta + g_n \cos\theta$, which is known as Chiral Equivalence Theorem and holds in the absence of physical magnetic charges He also showed that in the presence of physical magnetic charges, charge quantization condition applied not to separate products but to the combinations $(e_n g_m - e_m g_n)$ which must be integral multiples of 4π. Later

Zwanziger (1971) elegantly formulated the Local Lagrangian quantum field theory of electric and magnetic charges with the aid of a fixed arbitrary vector n to avoid complications of Dirac singularity line and obtained quantization of combination $(e_n g_m - g_n e_m)$ as a condition for the infinitesimal Poincaré-Lie algebra to be inte grated to a representation of finite Poincaré group

Here we propose to extend Zwanziger's (1971) work for spin-1/2 fields to the case of electric and magnetic charges carried by spin-zero fields like pions. The quantization condition that one would obtain in such a case is not very much obvious since the Lorentz transofrmation properties of spinor and boson fields differ. Considerable calculational differences are also encountered in obtaining expressions for various physical quantities. Here the usual technique is employed to obtain the canonical commutation relations, the energy momentum tensor, Lorentz transformation law and the Feyman rules. Our treatment for obtaining the commutation relations is, however, much more transparent and simpler than Zwanziger's.

In section 2 the local Lagrangian density involving two four-potentials A_μ and B_μ is introduced which yields Maxwells equations. In section 3 the Lagrangian density for electrically and magnetically charged boson fields is introduced and energy-momentum tensor calculated In section 4 the canonical commutation relations are found using axial gauge. In section 5 the law of transformation of field variables under infinitesimal change of Lorentz frame is found and quantization of $(e_n g_m - g_n e_m)$ is obtained on condition of its integrability to a finite Lorentz group In the last section the Feyman rules are noted for boson electrodynamics

2 Field equations and Lagrangian

The Maswell's equations are,

$$\partial_\mu F^{\mu\nu} = j_e^{\ \nu}, \quad \partial_\mu F^{d\mu\nu} = j_\xi^{\ \nu} \qquad \ldots \quad (2.1)$$

where electric and magnetic currents are conserved i e.

$$\partial . j_e = \partial\, j_\xi = 0 \qquad \ldots \quad (2.2)$$

The general solutions of 1st and 2nd Maxwell's equations are the ts

$$F = -(\partial \Lambda B)^d + (n.\partial)^{-1}(n \Lambda j_e) \qquad \ldots \quad (2.3a)$$

$$F^d = (\partial \Lambda B) + (n.\partial)^{-1}(n \Lambda j_e)^d \qquad \ldots \quad (2.3b)$$

and

$$F^d = (\partial \Lambda A)^d + (n.\partial)^{-1}(n \Lambda j_\xi) \qquad \ldots \quad (2.4a)$$

$$F = (\partial \Lambda A) - (n.\partial)^{-1}(n \Lambda j_\xi)^d \qquad \ldots \quad (2.4b)$$

respectively. Where n is an arbitrary fixed four-vector, $(n.\partial)^{-1}$ is an integral operator with kernel $(n\,\partial)^{-1}\,(x-y)$ satisfying $(n\,\partial)(n\,\partial)^{-1}(x)$ and A_μ and B_μ are two four-potentia .

Equations (2.3b) and (2.4b) give $n.F^d = n.(\partial\Lambda B)$, $n.F = n.(\partial\Lambda A)$ ... (2.5)

which yield.

$$F = \frac{1}{n^2}(\{n\Lambda[n.(\partial\Lambda A)]\}-\{n\Lambda[n.(\partial\Lambda B)]\}^d) \qquad \text{... (2.6a)}$$

$$F^d = \frac{1}{n^2}(\{n\Lambda[n.(\partial\Lambda A)]\}^d+\{n\Lambda[n.(\partial\Lambda B)]\}) \qquad \text{... (2.6b)}$$

Using these expressions, Maxwell's equations in terms of potentials are,

$$\frac{1}{n^2}(n.\partial n.\partial A^\mu - n.\partial\partial^\mu n.A - n^\mu n.\partial\partial.A - n^\mu\partial^2 n.A - n.\partial\epsilon^\mu{}_{\nu k\lambda}n^\nu\partial^k B^\lambda) = j_e{}^\mu \qquad (2.7a)$$

$$\frac{1}{n^2}(n.\partial n.\partial B^\mu - n.\partial\partial^\mu n.B - n^\mu n.\partial\partial.B - n^\mu\partial^2 n.B - n.\partial\epsilon^\mu{}_{\nu k\lambda}n^\nu\partial^k A^\lambda) = j_\zeta{}^\mu \qquad (2.7b)$$

These equations of motion follow from Lagrangian density

$$L = L_r + L_I \qquad (2.8)$$

where

$$\begin{aligned} L_r = & -\frac{1}{2n^2}[n.(\partial\Lambda A)].[n.(\partial\Lambda B)^d] \\ & +\frac{1}{2n^2}[n.(\partial\Lambda B)][n.(\partial\Lambda A)^d] \\ & -\frac{1}{2n^2}[n.(\partial\Lambda A)]^2 - \frac{1}{2n^2}[n.(\partial\Lambda B)]^2 \end{aligned} \qquad (2.9)$$

and

$$L_I = -j_e.A - j_g.B \qquad (2.10)$$

$$j_e{}^\nu = \sum_n ie_n[(\partial_\nu + ie_nA_\nu + i\zeta_nB_\nu)\phi_n{}^*\phi_n - \phi_n{}^*(\partial_\nu - ie_nA_\nu - i\zeta_nB_\nu)\phi_n] \qquad (2.11a)$$

$$j_g{}^\nu = \Sigma\, i\zeta_n[(\partial_\nu + ie_nA_\nu + i\zeta B_\nu)\phi_n{}^*\phi_n - \phi_n{}^*(\partial_\nu - ie_nA_\nu - i\zeta B_\nu)\phi_n] \qquad (2.11b)$$

where ϕ_n's are a set of spin-zero fields each carrying electric and magnetic charges e_n and ζ_n

It can be easily verified that these currents are conserved by virtue of equations of motion for the spin 0 fields.

3. Energy-momentum tensor

Now the total Lagrangian-density is,

$$L = L_r + L_M + L_I \qquad \text{... (3.1)}$$

where L_r is given by equation (2.9) and,

$$L_M+L_I$$
$$=\sum_n(\partial_\mu-ie_nA_\mu-ig_nB_\mu)\phi_n(x)\,(\partial_\mu+ie_nA_\mu+i\zeta B_\mu)\phi_n^*(x)-m_n^2\phi_n^*(x)\phi_n(x) \quad \ldots \quad (3.2)$$

The Lagrangian equations of motion for boson fields is,

$$[\{\partial_\lambda-(ie_nA_\lambda+i\zeta B_\lambda)\}^2+m_n^2]\phi_n(x)=0 \quad \ldots \quad (3\ 3)$$

The Lagrangian, (3.1) depends locally on fields Φ's *i e.*, A, B, ϕ_n and the fixed four-vector n, i e ,

$$L(x)=L(\Phi_\alpha(x),\,\partial_\alpha\Phi_\alpha(x),\,n) \quad \ldots \quad (3\ 4)$$

So one has in the standard way,

$$T^{\mu\nu}=\sum_\alpha\frac{\partial L}{\partial\,\partial_\mu\Phi_\alpha}\,\partial^\nu\Phi_\alpha-g^{\mu\nu}L \quad \ldots \quad (3.5)$$

with
$$\partial_\mu T^{\mu\nu}=0 \quad \ldots \quad (3.6)$$

However the peculiarities are observed with Lorentz transformation properties where one finds,

$$\partial_k M^{k\mu\nu}=(n^\mu\partial_n{}^\nu-n^\nu\partial_n{}^\mu) \quad \ldots \quad (3.7)$$

instead of being zero. Now define $M^{k\mu\nu}$ by,

$$M^{k\mu\nu}=\frac{\partial L}{\partial\partial_k\Phi_\alpha}\{(x^\mu\partial_\nu-x^\nu\partial_\mu)\Phi_\alpha+\Sigma^{\mu\nu}{}_{\alpha s}\Phi_\alpha\}-(x^\mu g^{\nu k}-x^\nu g^{\mu k})L \quad \ldots \quad (3.8)$$

It is known that $\Sigma^{\mu\nu}{}_{\alpha s}$ is zero for scalar fields, so that inserting equation (3.8) into (3 7) and using (3 5) one gets,

$$T^{\mu\nu}-T^{\nu\mu}+\partial_k\sum_\alpha\frac{\partial L}{\partial\partial_k\Phi_\alpha}\,\Sigma^{\mu\nu}{}_\alpha\,\phi_\alpha=(n^\mu\partial_n{}^\nu-n^\nu\partial_n{}^\mu)L \quad \ldots \quad (3\ 9)$$

Defining symmetric energy-momentum tensor $\theta_{\mu\nu}$ as

$$\theta_{\mu\nu}=T^{\mu\nu}+\tfrac{1}{2}\partial_k\left[\frac{\partial L}{\partial\partial_k\Phi_\alpha}\,\Sigma_\alpha{}^{\mu\nu}\phi_\alpha-\frac{\partial L}{\partial\partial_\mu\Phi_\alpha}\,\Sigma_\alpha{}^{k\nu}\Phi_\alpha-\frac{\partial L}{\partial\partial_\nu\Phi_\alpha}\,\Sigma_\alpha{}^{k\mu}\Phi_\alpha\right] \ldots \quad (3\ 10)$$

and using (3.9)
one finds

$$\theta_{\mu\nu}=\tfrac{1}{2}(T^{\mu\nu}+T^{\nu\mu})-\tfrac{1}{2}\partial_k\sum_\alpha\left(\frac{\partial L}{\partial\partial_\mu\Phi_\alpha}\,\Sigma_\alpha{}^{k\nu}\Phi_\alpha+\frac{\partial L}{\partial\partial_\nu\Phi_\alpha}\,\Sigma_\alpha{}^{k\mu}\Phi_\alpha\right.$$
$$+(n^\alpha\partial_n{}^\nu-n^\nu\partial_n{}^\mu)L \quad \ldots \quad (3\ 11)$$

Using L_r which alone contains n, one gets,

$$(n^\mu \partial_n{}^\nu - n^\nu \partial_n{}^\mu) L_r = -n\Lambda\{n\,[(n.\partial)^{-1} j_e \Lambda (n\,\partial)^{-1} j_\zeta]^d\} \quad \dots \quad (3.12)$$

so finally,

$$\theta_{\mu\nu} = \tfrac{1}{2}(T^{\mu\nu} + T^{\nu\mu}) - \tfrac{1}{2}\partial_k \sum_\alpha \left(\frac{\partial L}{\partial \partial_\mu \Phi_\alpha} \Sigma_\alpha{}^{k\nu} \Phi_\alpha + \frac{\partial L}{\partial \partial_\nu \Phi_\alpha} \Sigma_\alpha{}^{k\mu} \Phi_\alpha \right)$$

$$-n\Lambda\{n\,[(n.\partial)^{-1} j_e \Lambda (n.\partial)^{-1} j_\zeta\,]^d\} \quad \dots \quad (3.13)$$

which is symmetric except for the last term So

$$\frac{\mathrm{d}}{\mathrm{d}t} M^{\mu\nu} = \int (\theta^{\mu\nu} - \theta^{\nu\mu}) \mathrm{d}^3 x = -\int (n\Lambda\{n.[(n.\partial)^{-1} j_e \Lambda (n\,\partial)^{-1} j_\zeta\,]^d\})^{\mu\nu} \mathrm{d}^3 x \quad \dots \quad (3.14)$$

Using total Lagrangian (3.1) and (3 13) one gets,

$$\theta_{\mu\nu} = \tfrac{1}{2}(F.F + F^d\,F^d)^{\mu\nu} + \sum_n (\partial_\mu + ie_n A_\mu + i\zeta_n B_\mu)\phi_n{}^* \partial_\nu \phi_n$$

$$+ \sum_n (\partial_\mu - ie_n A_\mu - i\zeta_n B_\mu)\phi_n . \partial_\nu \phi_n{}^* - \sum_n g^{\mu\nu} [(\partial + ie_n A + ig_n B)\phi_n{}^* . (\partial - ie_n A$$

$$- i\zeta_n B)\phi_n] - n^\mu \{n\,[(n\,\partial)^{-1} . j_e \Lambda (n\,\partial)^{-1} j_\zeta]^d\}^\nu \quad \dots \quad (3.15)$$

4. Canonical commutation relations

To linearise the Lagrangian equation of motion (2 7) one imposes the axial gauge conditions

$$n\,A = n.B = 0 \quad \dots \quad (4.1)$$

The procedure is to add lagrangian density

$L_G = \frac{1}{2n^2}\{[\partial(n.A)]^2 + [\partial(n\,B)]^2\}$ to eqn (3.1) to obtain total Lagrarian density as,

$$L = L_r + L_G + L_M + L_I \quad \dots \quad (4.2)$$

Then

$$\theta_{\mu\nu} = \tfrac{1}{2}(F\,F + F^d\,F^d)^{\mu\nu} + \sum_n \{(\partial_\mu \zeta - ie_n A_\mu + i\zeta_n B_\mu)\phi_n{}^* \partial_\nu \phi_n$$

$$+ (\partial_\mu - ie_n A_\mu - i\zeta_n B_\mu)\phi_n \partial_\nu \phi_n{}^*$$

$$- g^{\mu\nu}[(\partial + ie_n A + i\zeta_n B)\phi_n{}^* . (\partial - ie_n A - i\zeta_n B)\phi_n]\}$$

$$+ \frac{1}{n^2}[\partial_\mu n.A \partial_\nu n\,A + \partial_\mu n.B \partial_\nu n\,B - g^{\mu\nu}\tfrac{1}{2}\{(\partial n.A)^2 + (\partial n.B)^2\}]$$

$$- n^\mu \{n.[(n.\partial)^{-1} j_e \Lambda (n.\partial)^{-1} j_\zeta]^d\}^\nu \quad \dots \quad (4.3)$$

Now we consider the question of equal-time commutation relations. Zwanziger (1971) has deduced this relation by calculating a set of canonical variable. It is much better to follow a method of Pierles to calculate the commutators, which is given below for our case. This method can also be used for spin-1/2 fields. We will take $n = (0, \mathbf{n})$ hereafter without loss of generality. From the Lagrangian one gets for four-potentials A and B

$$[A^{\mu}(t, x), B^{\nu}(t, y)] = i\epsilon_{k0}{}^{\mu\nu} n^{k}(n.\partial)^{-1}(x-y) \quad \ldots \quad (4.4a)$$

$$[B^{\mu}(t, x), B^{\nu}(t, y)] = [A^{\mu}(t, x), A^{\nu}(t.y)] = -i(g_0{}^{\mu}n^{\nu}+g_0{}^{\nu}n^{\mu})(n\,\partial)^{-1}(x-y) \quad (4.4b)$$

All the equations here-to-fore are invariant under simultaneous substitutions,

$$(e_n g_n) \rightarrow (e_n \cos\theta - g_n \sin\theta,\ e_n \sin\theta + g_n \cos\theta)$$

$$(A, B) \rightarrow (\cos\theta\, A - \sin\theta . B,\ A\ \sin\theta + B.\cos\theta)$$

or equivalently $(F, F^d) \rightarrow (\cos\theta . F - \sin\theta . F^d, \cos\theta\, . F^d + \sin\theta\, F)$.

Now the quantization of ϕ_n's is governed by the commutations

$$[\phi_n(x),\ \phi_n(y)] = 0 = [\phi_n{}^*(x), \phi_n{}^*(y)] \quad \ldots \quad (4.5a)$$

and $$[\pi_n(x, t), \phi_n(y, t)] = [\pi_n{}^*(x, t), \phi_n{}^*(y, t)] = -i\delta^3(x-y) \quad \ldots \quad (4.5b)$$

with $$\pi_n(x) = [\partial_0(x) + ieA_0(x) + i\zeta B_0(x)]\phi_n{}^* \quad \ldots \quad (4.6a)$$

and $$\pi_n{}^*(x) = [\partial_0(x) - ieA_0(x) - i\zeta B_0(x)]\phi_n \quad \ldots \quad (4.6b)$$

The ϕ_n's commute with A and B.

From equations (2.3) and (2.4) the electromagnetic fields can be written as,

$$-F^{d0i} = \tfrac{1}{2}\epsilon_{ijk}F^{jk} = \boldsymbol{H} = \nabla \times A + (n.\nabla)^{-1}\mathbf{n}\,\rho_{\zeta} \equiv H_i \quad \ldots \quad (4.7a)$$

$$-F^{0i} = \tfrac{1}{2}\,\epsilon_{ijk}F^{djk} = \boldsymbol{E} = -\nabla \times B + (n\,\nabla)^{-1}\mathbf{n}\rho_e = E_i \quad \ldots \quad (4.7b)$$

where,

$$\rho_e = \sum_n ie_n[(\partial_0 + ie_nA_0 + i\zeta_nB_0)\phi_n{}^*\ \phi_n - \phi_n{}^*(\partial_0 - ie_nA_0 - i\zeta_nB_0)\phi_n]$$

$$\rho_{\zeta} = \sum_n ig_n[(\partial_0 + ie_nA_0 + i\zeta_nB_0)\phi_n\ \phi_n{}^* - \phi_n{}^*(\partial_0 - ie_nA_0 - i\zeta_nB_0)\phi_n]$$

which lead to following commutation relation,

$$[E_i(x), H_j(y)] = i\epsilon_{ij}\ \nabla_k\delta(x-y) \quad \ldots \quad (4.8a)$$

$$[E_i(x), E_j(y)] = [H_i(x), H_j(y)] = 0 \quad \ldots \quad (4.8b)$$

$$[\boldsymbol{E}(x), \phi_n(y)] = -e_n\mathbf{n}(n.\nabla)^{-1}(x-y)\phi_n(y) \quad \ldots \quad (4.9a)$$

$$[\boldsymbol{H}(x), \phi_n(y)] = -g_n\mathbf{n}(n\,\nabla)^{-1}(x-y)\phi_n(y) \quad \ldots \quad (4.9b)$$

From equation (4 3) one easily gets, the Hamiltonian density θ^{00} to be,

$$\theta_{00} = \tfrac{1}{2}[(\nabla\times A)^2+(\nabla\times B)^2]$$

$$-\frac{1}{2n^2}[(\boldsymbol{n}\,\nabla A^0-\boldsymbol{n}\cdot\nabla\times B)^2+(\boldsymbol{n}\,\nabla B^0+\boldsymbol{n}\cdot\nabla\times A)^2+(\nabla\boldsymbol{n}.A)^2+(\nabla\boldsymbol{n}.B)^2]$$

$$-\boldsymbol{n}\,\nabla[A^0(\boldsymbol{n}\cdot\nabla)^{-1}\rho_e+B^0(\boldsymbol{n}\cdot\nabla)^{-1}\rho_\zeta]$$

$$+\Sigma\{(\nabla+ie_n\boldsymbol{A}+i\zeta\boldsymbol{B})\phi_n^*.(\nabla ie_n\boldsymbol{A}-i\zeta_n\boldsymbol{B})\phi_n+\phi_n^*\dot{\phi}_n+m_n^2\phi_n^*.\phi_n\} \tag{4.10}$$

and

$$\theta_{0i} - (\nabla\times\boldsymbol{A})\times(\nabla\times\boldsymbol{B})-\frac{1}{n^2}[\boldsymbol{n}\,\nabla A^0-\boldsymbol{n}\cdot\nabla\times B]\nabla\boldsymbol{n}\cdot\boldsymbol{A}$$

$$-\frac{1}{n_2}[\boldsymbol{n}\,\nabla B^0+\boldsymbol{n}\,\nabla\times A]\nabla\boldsymbol{n}\,\boldsymbol{B}-\boldsymbol{n}\,\nabla[\boldsymbol{A}(n\,\Delta)^{-1}\rho_e+\boldsymbol{B}(n\,\Delta)^{-1}\rho_\zeta]$$

$$+\sum_n\{(\Delta_i+ie_n\boldsymbol{A}_i+i\zeta_n\boldsymbol{B}_i)\phi_n^*.\phi_n+(\nabla_i-ie_n\boldsymbol{A}_i-i\zeta_n\boldsymbol{B}_i)\phi_n.\phi_n^*\} \tag{4.11}$$

5. Charge quantization

The infinitesimal transformations law for various operators are given by their equal-time commutation relation with $M_{\mu\nu}$ In the present case it is found.

$$-i[\phi_n, M_{\mu\nu}] = [(x^\mu\partial^\nu-x^\nu\partial^\mu)+i\eta_n^{\mu\nu}]\phi_n \qquad \ldots \quad (5.1\text{a})$$

$$-i[A_k, M_{\mu\nu}] = (x_\mu\partial_\nu-x_\nu\partial_\mu)A^k+(g^{k\mu}A^\nu-g^{k\nu}A^\mu)$$

$$-\partial_k\eta_e^{\mu\nu}-(n.\partial)^{-2}[n^\mu(n\Lambda j\zeta)^{d\nu k}-n^\nu(n\Lambda j_\zeta)^{d\mu k}] \qquad \ldots \quad (5.1\text{b})$$

$$-i[B_k, M_{\mu\nu}] = (x_\mu\partial_\nu-x_\nu\partial_\mu)B_k+(g^{k\mu}B^\nu-g^{k\nu}B^\mu)-\partial_k\eta\zeta^{\mu\nu}$$

$$-(n.\partial)^{-2}[n^\mu(n\Lambda j_e)^{d\nu k}-n^\nu(n\Lambda j_\zeta)^{d\mu k}] \qquad \ldots \quad (5.1\text{c})$$

where $\eta_e^{\mu\nu} = (n.\partial)^{-1}(n\Lambda A)^{\mu\nu}$, $\eta_\zeta^{\mu\nu} = (n.\partial)^{-1}(n\,\Lambda B)^{\mu\nu}$, $\eta_n^{\mu\nu} = e_n\eta_e^{\mu\nu}+\zeta_n\eta_\zeta^{\mu\nu}$

From the equation (5.1b) and (5.1c) it is evident that $n.A$ and $n\,B$ transform like scalars i e

$$-i[n.A, M^{\mu\nu}] = (x^\mu\partial_\nu-x^\nu\partial_\mu)n\,A \text{ and } A\to B \qquad \ldots \quad (5\,2)$$

one also finds $I_n^\mu = (\partial_\mu\phi_n^*\,\phi_n-\phi_n^*\partial_\mu\phi_n)$ transforms like a vector i e.

$$-i[I_n^k, M^{\mu\nu}] = (x_\mu\partial_\nu-x_\nu\partial_\mu)I_n^k+g^{\mu k}I_n^\nu-g^{\nu k}I_n^\nu \qquad \ldots \quad (5.3)$$

and $-i[F^{k\lambda}, M^{\mu\nu}] = (x^\mu\partial_\nu-x^\nu\partial_\mu)F_{k\lambda}+(g^{k\mu}F^{\nu\lambda}-g^{k\nu}F^{\mu\lambda})+(g^{\lambda\mu}F^{k\nu}-g^{\lambda\nu}F^{k\mu})$ (5.4)

The finite transformation law of $n.A$, $n.B$, I_n^k and $F^{\mu\nu}$, A^μ are,

$$n.A^\Lambda(x) = U(\Lambda)n.A(x)U^{-1}(\Lambda) = n.A(\Lambda x) \text{ and } A\to B \qquad \ldots \quad (5.5\text{a})$$

$$I_n^{\mu\Lambda}(x) = U(\Lambda)I_n^{\mu}U^{-1}(\Lambda) = \Lambda_\nu^{-1\mu}I_n^{\nu}(\Lambda x) \quad \dots \quad (5.5b)$$

$$F^{\Lambda}{}_{\mu\nu}(x) = U(\Lambda)F_{\mu\nu}{}^{\Lambda}(x)U^{-1}(\Lambda) = \Lambda_\sigma^{-1\mu}\Lambda_\tau^{-1\nu}F^{\sigma\tau}(\Lambda x) \quad \dots \quad (5.5c)$$

$$A_\mu{}^{\Lambda}(x) = U(\Lambda)A^{\mu}(x)U^{-1}(\Lambda) = \Lambda^{-1}{}_\nu{}^{\mu}A^{\nu}(\Lambda x)+\partial_\mu\eta_e{}^{\Lambda}(x)$$
$$+[n.\partial(\Lambda^{-1}n.\partial)]^{-1}\epsilon^{\mu}{}_{\rho\sigma\tau}n^{\rho}(\Lambda^{-1}n)^{\nu}j_e{}^{\Lambda\,\tau} \quad \dots \quad (5.5d)$$

$$B^{\Lambda\mu}(x) = U(\Lambda)B^{\mu}U^{-1}(\Lambda) = \Lambda^{-1}{}_\nu{}^{\mu}B^{\nu}(\Lambda x)+\partial_\mu\eta_\zeta{}^{\Lambda}(x)$$
$$-[(n.\partial)(\Lambda^{-1}n.\partial)]^{-1}\epsilon^{\mu}{}_{\rho\sigma\tau}n^{\rho}(\Lambda^{-1}n)^{\sigma}j_e\Lambda T \quad \dots \quad (5.5e)$$

where $\qquad U(\Lambda) = \exp(M.\Omega),$

$$\eta^{\Lambda}{}_e(x) = \int (n\,\partial)^{-1}(x-y)(n-\Lambda n)A(\Lambda y)\mathrm{d}^4y \quad \dots \quad (5.5f)$$

$$\eta^{\Lambda}{}_\zeta(x) = \int (n\,\partial)^{-1}(x-y)(n-\Lambda n)B(\Lambda y)\mathrm{d}^4y \quad \dots \quad (5.5g)$$

and the kernel

$$(n\,\partial\ \ n'\,\partial)^{-1} = \tfrac{1}{4}\int \delta^4(\boldsymbol{x}-\boldsymbol{n}s-\boldsymbol{n}t)|\,\epsilon(s)\,\epsilon(t)+1\,|\mathrm{d}s\mathrm{d}t. \quad \dots \quad (5.6)$$

To verify group multiplication property one has to use the property.

$$U(\Lambda_2)\eta_{\substack{e\\ \zeta}}^{\Lambda_1}U^{-1}(\Lambda_2) = \eta_{\substack{e\\ \zeta}}^{\Lambda 21}-\eta_{\substack{e\\ \zeta}}^{\Lambda 2}(\Lambda_1 x) \pm [(n\,\partial\,n_1\,\partial\,n_{21}.\partial)^{-1}\epsilon_{k\lambda\mu\nu}n^k n_1{}^{\lambda}n_{21}{}^{\mu}j_{\zeta}^{\Lambda 21}(x)] \quad \dots \quad (5\,7)$$

Next we consider the transformation law for ϕ_n which takes tho form,

$$\phi^{\Lambda}{}_n(x) = U(\Lambda)\phi_n(x)U^{-1}(\Lambda) = \phi_n(\Lambda x)\exp[-i\eta^{\Lambda}{}_n(x)] \quad \dots \quad (5.8)$$

where $\qquad \eta^{\Lambda}{}_n = e_n\eta^{\Lambda}{}_e(x)+\zeta_n\eta^{\Lambda}{}_\zeta(x)$

as can be seen from the commutation relation equation (5.1a). Now we consider the group multiplication property for rotations We get, from (5 7) and (5.8)

$$U(R_2)U(R_1)\phi_n(x)U^{-1}(R_1)U^{-1}(R_2) = \phi_n(R_{21}x)\exp[-i\eta_n{}^{R_2}(R_1x)]$$
$$\exp\{-i[\eta_n{}^{R_{21}}(x)-\eta_n{}^{R_2}(R_1\,x)-\delta(\boldsymbol{x})]\} \quad \dots \quad (5.9)$$

where $\qquad \delta(\boldsymbol{x}) = \boldsymbol{n}\cdot\boldsymbol{n}_1\times\boldsymbol{n}_{21}(n.\nabla n_1\,\nabla n_{21}.\nabla)^{-1}(e_n\rho_\zeta{}^{R_{21}}-\zeta_n\rho_e{}^{R_{21}}) \quad \dots \quad (5.10)$

and $\qquad R_{21} = R_2R_1, \quad \boldsymbol{n}_1 = R_1{}^{-1}\boldsymbol{n},\ \boldsymbol{n}_{21} = R_{21}{}^{-1}\boldsymbol{n}$

Since $\delta(x)$ commutes with other factors in the exponent and $\eta_n{}^{R_2}(R_1x)$ with $\eta_n{}^{R_{21}}(x)$ one gets,

$$U(R_2)U(R_1)\phi_n(x)U^{-1}(R_1)U^{-1}(R_2) = U(R_{21})\phi_nU^{-1}(R_{21})e^{i\delta(x)} \quad \dots \quad (5.11)$$

So for the group property to be satisfied $ie^{\delta(x)}$ must be unity which means that eigenvalues of $\delta(x)$ must be integral multiple of 2π.

Now,

$$[\delta(x), \phi_m{}^{R_{21}}(y)] = -(e_n g_m - e_m g_n)\boldsymbol{n}_1\cdot\boldsymbol{n}_1\times\boldsymbol{n}_{21}(n.\nabla n_1.\nabla n_{21}.\nabla)^{-1}(x-y)\phi_m{}^{R_{21}}(y) \quad \ldots \quad (5.12)$$

So with

$$\boldsymbol{n}\cdot\boldsymbol{n}_1\times\boldsymbol{n}_{21}(n.\nabla n_1.\nabla n_{21}.\nabla)^{-1}(x) = \boldsymbol{n}\cdot\boldsymbol{n}_1\times\boldsymbol{n}_{21}\tfrac{1}{8}\int\delta^3(\boldsymbol{x}-\boldsymbol{n}s-\boldsymbol{n}_1 t-\boldsymbol{n}_{21}u)$$
$$\times[\epsilon(s)\epsilon(t)\epsilon(u)+\epsilon(s)+\epsilon(t)+\epsilon(u)]\mathrm{d}s\mathrm{d}t\mathrm{d}u$$
$$= \tfrac{1}{8}[\epsilon(s_0)\epsilon(t_0)\epsilon(u_0)+\epsilon(s_0)+\epsilon(t_0)+\epsilon(u_0)] \quad \ldots \quad (5\ 13)$$

where $s_0 = \boldsymbol{x}\cdot\boldsymbol{n}\times\boldsymbol{n}_1$, $t_0 = \boldsymbol{x}\cdot\boldsymbol{n}_1\times\boldsymbol{n}_{21}$, $u_0 = \boldsymbol{x}\cdot\boldsymbol{n}_{21}\times\boldsymbol{n}$

It is easy to see that $1/8[\epsilon(s_0)\epsilon(t_0)\epsilon(u_0)+\epsilon(s_0)+\epsilon(t_0)+\epsilon(u_0)]$ takes values or 0 $\pm 1/2$ depending on either one or two of s_0, t_0 and u_0 is negative or none or all negative respectively. It is evident from (5.12) that $\phi_m{}^{R_{21}}$ connects eigenstates of $\delta(x)$ whose eigenvalues differ by 0 or $\pm 1/2$ $(e_n g_m - g_n e_m)$. Hence for group multiplication property to be satisfied,

$$(e_n g_m - g_n e_m)/4\pi = C_{nm} \quad \ldots \quad (5.14)$$

where C_{nm} is an integer. Thus the charge quantizaton obtained for spinor case is also here reproduced, as a condition that infinitesimal Lorentz transformation may be integrated to give finite Lorentz group.

6. Feynman Rules

The charged particle propagators are as usual,

$$-i < T\phi_n(x)\phi_n{}^*(y) > = \Delta_F(x-y,\ m) \quad \ldots \quad (6\ 1)$$

where $\Delta_F(x, m)$ satisfies, $(-\partial^2 - m^2)\Delta_F(x,\ m) = \delta^4(x)$

Putting $V_\mu{}^a = (A_\mu, B_\mu)$, $(a = 1, 2)$ … (6.2)

one gets the photon propagator to be,

$$-i < TV^a{}_\mu(x)V^b{}_\mu(y) > = \{[-g_{\mu\nu}+(\partial_\mu n_\nu + n_\mu\partial_\nu)(n\ \partial)^{-1}]\delta^{ab}$$
$$-\epsilon_{\mu\nu\sigma\tau}n^\sigma\partial^\tau(n.\partial)^{-1}]\epsilon_{ab}\}\Delta_F(x-y, 0) \quad \ldots \quad (6.3)$$

One also has to replace vortices $e(p+p')_\mu$ and $-2e^2 g_{\mu\nu}$ of ordinary spin zero electrodynamics by $q^a{}_n(p+p')_\mu$ and $-2(q_n{}^a)^2 g_{\mu\nu}$ where charge vector $q_n{}^a$ is given by,

$$q_n{}^a = (e_n, \zeta_n) \quad \ldots \quad (6.4)$$

It is of course quite known that the S operator, given by,

$$s = T\exp[i\int H_I(x)\mathrm{d}^4x] \quad \ldots \quad (6.5)$$

where T is time ordered product, cannot be expanded into an infinite series because of the large magnetic coupling constant.

Conclusion

We have been able to show that the Chiral Invariant combination $(e_m g_n - e_n g_m)$ not eg need be quantized as a condition of consistence of the field theoretic formulaton of spin-zero meson carrying electric and magnetic charges. It is of course evident that with a little more calculational complications theory can be extended to the case of spin-1 fields.

If the magnetic charge is non-zero, the value it has is reciprocal to the electrical charge and turns out to be too high. One charged particle carrying charges $e_1 = e$, $g_1 = 0$ leads to another charged particle to carry charges $e_2 = e$, $g_2 = 4\pi/e$. The second particle will have super-strong interactions with electromagnetic field with $g^2/4\pi = 137$. It may be easier to argue that for pi-mesons such interactions are presumably ruled out by present day high energy experiments.

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